An Analytical Approach for the Evaluation of the Optimal Combustion Phase in Spark Ignition Engines

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It is well known that the spark advance is one of the most important parameters influencing the efficiency of a spark ignition engine. A change in this parameter causes a shift in the combustion phase, whose optimal position, with respect to the piston motion, implies the maximum brake mean effective pressure for given operative conditions. The best spark timing is usually estimated by means of experimental trials on the engine test bed or by means of thermodynamic simulations of the engine cycle. In this work, instead, the authors developed, under some simplifying hypothesis, an original theoretical formulation for the estimation of the optimal combustion phase. The most significant parameters involved with the combustion phase are taken into consideration; in particular, the influence of the combustion duration, of the heat release law, of the heat transfer to the combustion chamber walls, and of the mechanical friction losses is evaluated. The theoretical conclusion, experimentally proven by many authors, is that the central point of the combustion phase (known as the location of the 50% of mass fraction burnt, here called MFB50) must be delayed with respect to the top dead center as a consequence of both heat exchange between gas and chamber walls and friction losses.

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1 Introduction

The phasing of the combustion process with respect to the piston motion is one of the most important parameters influencing the torque provided by a spark ignition (SI) engine. Since this combustion phase depends on the spark ignition timing, located at the so called ignition angle, the control upon this angle is very important to obtain the best performance in every operative condition.

Two different approaches are commonly followed for the determination of the maximum brake torque (MBT) spark timing: The first is based on experimental trials at the engine test bed, which can be performed by either maximizing the engine torque (see, e.g., Refs. [1–3]) or setting a "combustion phase indicator" (which is a parameter derived from in-cylinder pressure analysis and assumes reference fixed values when the combustion timing is optimal [4]) to its best value [5–7]. The second approach, instead, proceeds by means of simulations based on a thermodynamic model of the engine [8–11], which, endowed of appropriate submodels for combustion [12], heat transfer [13], and friction loss [8,14,15] modeling, allow the brake mean effective pressure (bmep) estimation once the spark advance is fixed. For each engine operative condition, the best combustion phase can be found by means of successive trials. In this paper, instead, the authors propose an original theoretical approach to the problem of the determination of the optimal combustion phase in spark ignition engines.

With reference to the working cycle of the engine (compression and expansion strokes during a crankshaft rotation from −180 deg to +180 deg with respect to the top dead center (TDC)), the angular phase of the heat introduction Q_1 that grants the maximum torque first depends on the way the combustion takes place (i.e., on the heat release law) and then on the effects of the heat Q_1 upon the various thermodynamic variables involved.

The theoretical approach proposed can be carried out at different approximation levels with respect to the following hypothesis:

(a) unsteady, ideal, zero-dimensional evolution of a perfect gas in the combustion chamber
(b) adiabatic engine (i.e., no heat exchange between the gas and the chamber walls)
(c) constant specific heat capacity (c_v) of the gas
(d) instantaneous combustion
(e) combustion length $\vartheta_c=\vartheta_b-\vartheta_a$, being ϑ_a and ϑ_b the crank angular positions at the starting and ending of the combustion process
(f) presence of heat transfer between the gas and the chamber walls
(g) presence of friction losses

2 Gas Temperature Trend During Combustion

Considering that previous hypotheses (a)–(c) and (e) are true, it is quite simple to evaluate the gas temperature trend versus the crank angular position ϑ. Ignoring the gas speed and the subsequent viscous friction losses, it is possible to write the first and the second laws of thermodynamics during the infinitesimal combustion time dt, corresponding to the crank rotation angle $d\vartheta$, in which the gas receives the combustion specific heat dQ_{in}:

$$dQ_{\text{in}} = c_v dT + p dv = c_v dT + RT\frac{dv}{v} = TdS \qquad (1)$$

where T, v, p, S, c_v, and R are the gas temperature, specific volume, pressure, specific entropy, constant volume specific heat, and gas constant, respectively.

Assuming that

$$\varphi(\vartheta) = \frac{R}{c_v}\frac{1}{v}\frac{dv}{d\vartheta} = \frac{k-1}{v}\frac{dv}{d\vartheta}$$

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$$\psi(\vartheta) = -\frac{1}{c_v}\frac{dQ_{\text{in}}}{d\vartheta}$$

where k is the isentropic coefficient equal to the ratio between the constant pressure (c_p) and constant volume (c_v) specific heat, expression (1) assumes the form of a linear differential equation

$$\frac{dT}{d\vartheta} + T \cdot \varphi(\vartheta) + \psi(\vartheta) = 0 \tag{2}$$

whose integral is

$$T = e^{-\int \varphi(\vartheta) d\vartheta}\left[\text{const} - \int \psi(\vartheta) e^{\int \varphi(\vartheta) d\vartheta} d\vartheta\right]$$

The solution is then

$$Tv^{k-1} = \text{const} - \int \psi(\vartheta) \cdot v^{k-1} d\vartheta$$

which applied from the start ϑ_a to the end ϑ_b of the combustion process gives

$$T_b v_b^{k-1} = T_a v_a^{k-1} + \frac{1}{c_v}\int_{\vartheta_a}^{\vartheta_b} v^{k-1}\frac{dQ_{\text{in}}}{d\vartheta} d\vartheta = \zeta_b \tag{3}$$

From Eq. (1), the whole specific entropy increase ΔS_{ab}, corresponding to heat introduction $Q_1 = \int_{\vartheta_a}^{\vartheta_b} dQ_{\text{in}}$, is

$$\begin{aligned}\Delta S_{ab} &= \int_a^b \left[c_v \frac{dT}{T} + R\frac{dv}{v}\right] = c_v\left[\ln\frac{T_b}{T_a} + \frac{R}{c_v}\ln\frac{v_b}{v_a}\right] \\ &= c_v \ln\frac{T_b v_b^{k-1}}{T_a v_a^{k-1}} = c_v \ln\frac{\zeta_b}{\zeta_a} \\ &= c_v \ln\left[1 + \frac{1}{c_v T_a v_a^{k-1}}\int_{\vartheta_a}^{\vartheta_b} v^{k-1}\frac{dQ_{\text{in}}}{d\vartheta} d\vartheta\right] \\ &= c_v \ln\left[1 + \frac{1}{c_v T_M v_M^{k-1}}\int_{\vartheta_a}^{\vartheta_b} v^{k-1}\frac{dQ_{\text{in}}}{d\vartheta} d\vartheta\right]\end{aligned} \tag{4}$$

where T_M and v_M are the gas temperature and specific volume when the piston is at the bottom dead center (BDC) related to T_a and v_a by the isentropic law; these variables are independent of the combustion phase length and position, i.e., from ϑ_a and ϑ_b.

The minimum entropy increase ΔS_{ab} corresponds to the maximum efficiency of the thermodynamic cycle if compression and expansion phases are adiabatic; in fact, in this case, $-\Delta S_{ab}$ is also the entropy decrease corresponding to the heat subtraction Q_2, at constant volume v_M, needed to close the thermodynamic cycle of the gas.

With reference to the thermodynamic cycle (A_0EFGHH_0) of Fig. 1, assuming that $x(\vartheta) = Q_{\text{in}}(\vartheta)/Q_1$ is the heat fraction released during the crank rotation from ϑ_a to ϑ (being $\vartheta_a < \vartheta < \vartheta_b$) with respect to the total $Q_1 = Q_{\text{in}}(\vartheta_b)$,

$$Q_2 = c_v T_{A_0}(e^{\Delta S_{ab}/c_v} - 1) \tag{5}$$

$$\Rightarrow \frac{\Delta S_{ab}}{c_v} = \ln\left(1 + \frac{Q_2}{c_v T_{A_0}}\right) \tag{6}$$

Then, from relation (4), it follows that

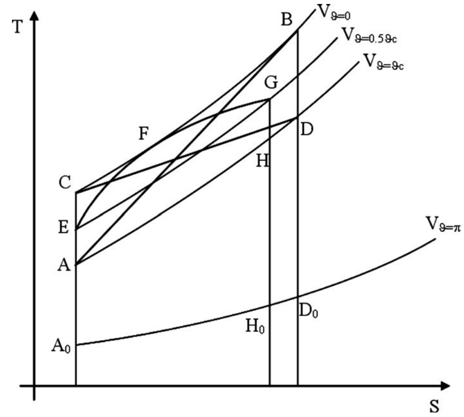

Fig. 1 Thermodynamic cycle with noninstantaneous combustion (AB, CD, and EFG have the same duration)

$$\begin{aligned}\frac{\Delta S_{ab}}{c_v} &= \ln\left[1 + \frac{1}{c_v T_C}\int_{\vartheta_a}^{\vartheta_b}\left(\frac{v}{v_C}\right)^{k-1}\frac{dQ}{d\vartheta} d\vartheta\right] \\ &= \ln\left[1 + \frac{Q_1}{c_v T_C}\int_{\vartheta_a}^{\vartheta_b}\left(\frac{v}{v_C}\right)^{k-1}\frac{dx}{d\vartheta} d\vartheta\right] \\ &\Rightarrow \frac{Q_2}{c_v T_{A_0}} = \frac{Q_1}{c_v T_C}\int_{\vartheta_a}^{\vartheta_b}\left(\frac{v}{v_m}\right)^{k-1}\frac{dx}{d\vartheta} d\vartheta\end{aligned} \tag{7}$$

$$1 - \eta = \frac{Q_2}{Q_1} = \frac{T_{A_0}}{T_C}\int_{\vartheta_a}^{\vartheta_b}\left(\frac{v}{v_m}\right)^{k-1}\frac{dx}{d\vartheta} d\vartheta = \frac{1}{\rho_0^{k-1}}\int_{\vartheta_a}^{\vartheta_b}\left(\frac{v}{v_m}\right)^{k-1}\frac{dx}{d\vartheta} d\vartheta$$

where $v_m = v_C$ is the gas specific volume at TDC, v_M is the gas specific volume at BDC, $\rho_0 = V_M/V_m$ is the volumetric compression ratio, and η is the thermodynamic cycle efficiency.

The above relations confirm that the combustion with a finite time ($\vartheta_c = \vartheta_b - \vartheta_a$), involving gas specific volumes higher than the TDC one v_m, leads to a lower efficiency than the instantaneous combustion, in which $v = v_m$, hence

$$1 - \eta = \frac{1}{\rho_0^{k-1}}$$

being, in this case,

$$\int_{\vartheta_a}^{\vartheta_b}\left(\frac{v}{v_m}\right)^{k-1}\frac{dx}{d\vartheta} d\vartheta = \int_{\vartheta_a}^{\vartheta_b}\frac{dx}{d\vartheta} d\vartheta = 1$$

3 Optimal Combustion Phase in the Adiabatic Engine

In the hypothesis d with instantaneous combustion,

$$\vartheta_a = \vartheta_b \Rightarrow \vartheta_c = 0$$

$$v = v_a = v_b = \text{const}$$

relation (6) yields

$$\Delta S_{ab} = c_v \ln\left(1 + \frac{Q_1}{c_v T_a}\right) \tag{8}$$

The entropy increase ΔS_{ab} reaches the minimum when T_a reaches the maximum, which happens at TDC ($T_A = T_C$ in Fig. 1).

When the combustion takes place during a finite phase $\vartheta_c = \vartheta_b - \vartheta_a$, the evaluation of ΔS_{ab} by means of relation (4) requires

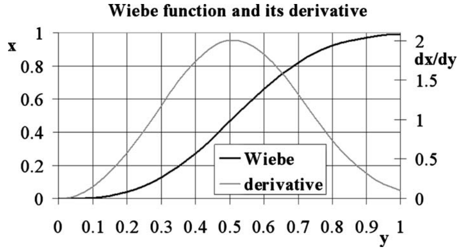

Fig. 2 The Wiebe function and its derivative, which is not a symmetric curve

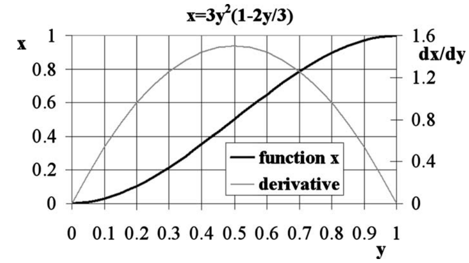

Fig. 3 The function (14) and its derivative

the knowledge of the two functions $V=V(\vartheta)$ and $dQ_{\text{in}}(\vartheta)/d\vartheta = Q_1(dx/d\vartheta)$.

Assuming that $V_0=V_M-V_m$ is the engine displacement, $\mu = r/c$ the ratio between the connecting rod length (r) and the crank radius (c), and ρ the compression ratio, the first function can be evaluated as follows:

$$V(\vartheta) = V_0\left[\frac{1}{\rho-1} + \frac{\mu}{2}\left(1 + \frac{1-\cos\vartheta}{\mu} - \sqrt{1 - \frac{sen^2\vartheta}{\mu^2}}\right)\right] \quad (9)$$

which is a symmetrical function with respect to the TDC position $\vartheta=0$.

Concerning the "heat release" function $Q_{\text{in}}(\vartheta)$ and its derivative $dQ_{\text{in}}(\vartheta)/d\vartheta$, it is opportune to evaluate some specific quantities such as $x=Q_{\text{in}}(\vartheta)/Q_1$ (fraction of heat released) and $y=(\vartheta - \vartheta_a)/\vartheta_c$ (fraction of combustion arc).

The function $x=x(y)$ has two constrained points, the origin O (0; 0) and the final point P (1; 1), because at the beginning of combustion ($\vartheta=\vartheta_a; y=0$), there is no heat released ($x=0$), while at the end ($\vartheta=\vartheta_b; y=1$), all the heat Q_1 has been released ($x=1$).

The Wiebe function (see Fig. 2), commonly used to model the heat released by the combustion in a SI engine, is

$$x = 1 - e^{-5y^3} \quad (10)$$

$$\frac{dx}{dy} = 15y^2(1-x) \quad (11)$$

The starting point of this function is ($y=0; x=0$) and for $y=1$, x is very close to unit (0.99) so it almost respects the two conditions mentioned above, while derivative (11) is not symmetric with respect to abscissa $y=0.5$, i.e., to the middle of the combustion arc.

A more simple function is

$$x = y \quad (12)$$

$$\frac{dx}{dy} = 1 \quad (13)$$

which have the extreme points O ($x=0; y=0$) and P ($x=1; y=1$), and a symmetric derivative.

The constant derivative (which means constant combustion velocity) is not plausible in a spark ignition engine; hence a better approximation is obtained by the following polynomial function:

$$x = 3y^2\left(1 - \tfrac{2}{3}y\right) \quad (14)$$

$$\frac{dx}{dy} = 6y(1-y) \quad (15)$$

which, as shown in Fig. 3, passes through the extreme points O ($x=0; y=0$) and P ($x=1; y=1$) and has a symmetric derivative (combustion speed with parabolic trend).

With respect to function (13), function (15) (as well as the classical Wiebe function (11)) has a more reasonable characteristic: The combustion rate is zero at the beginning ($y=0$) and at the end of the combustion process ($y=1$).

The function V^{k-1} is symmetric with respect to the TDC ($\vartheta=0$); if the function $dQ_{\text{in}}/d\vartheta$ (that is, dx/dy) is also symmetric with respect to the middle of the combustion angle ($y=0.5$), then, in Fig. 1, the two combustions AB, entirely located before the TDC ($\vartheta_a=-\vartheta_c; \vartheta_b=0$), and CD, entirely located after the TDC ($\vartheta_a=0; \vartheta_b=\vartheta_c$), produce the same entropy increase ΔS_{ab} (see Eqs. (3) and (4) and Fig. 4).

The combustion EFG (as shown in Figs. 1 and 4), which is symmetrically located with respect to the TDC ($\vartheta_a=-\vartheta_b=-0.5\vartheta_c$), represents instead the optimum because it produces the minimum entropy increase: The function $dQ_{\text{in}}/d\vartheta$, in fact, is the same as before (combustions AB and CD) but the function V^{k-1} has a smaller average value (the piston position, on the average, is closer to the TDC than before). It is important to notice that, as shown in Eqs. (4) and (7), the optimum condition does not depend on the whole heat released Q_1 but rather on the heat release law as a function of the time or crank angle (CA) rotation. An analytical approach to the determination of the optimal combustion phase for the adiabatic and frictionless engine is shown in the Appendix, also for the case of a not symmetrical heat release rate function.

4 Optimal Combustion Phase Taking Into Account Heat Exchanges and Friction Losses

Taking into account the heat exchanges between in-cylinder gas at temperature T and combustion chamber walls at temperature T_w, according to the hypothesis f in Sec. 1, Eq. (1) becomes

$$dQ_{\text{in}} - dQ_w = c_v dT + RT\frac{dv}{v} \quad (16)$$

where dQ_w is the specific heat subtracted, in the time interval dt, from the gas by the chamber walls, whose mean temperature T_w can be assumed to be constant:

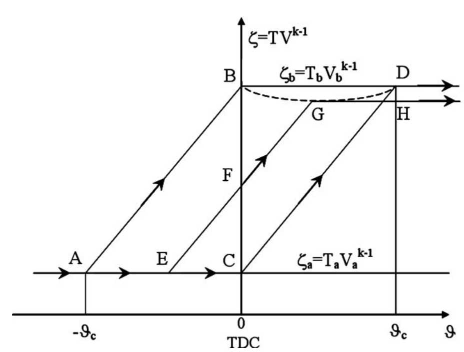

Fig. 4 Progress of $\zeta = TV^{k-1}$ as a function of the crank position ϑ for three combustions with different phase and same heat release law (see Eq. (3))

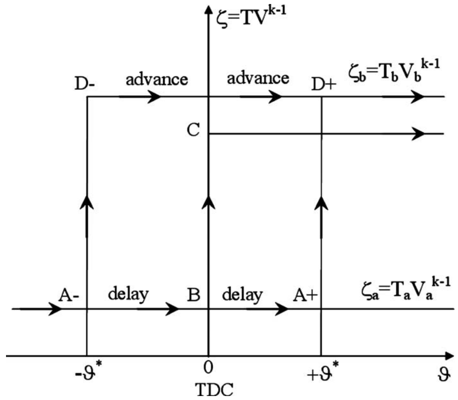

Fig. 5 Progress of $\zeta = TV^{k-1}$ as a function of the crank position ϑ for three instantaneous combustions with different phase

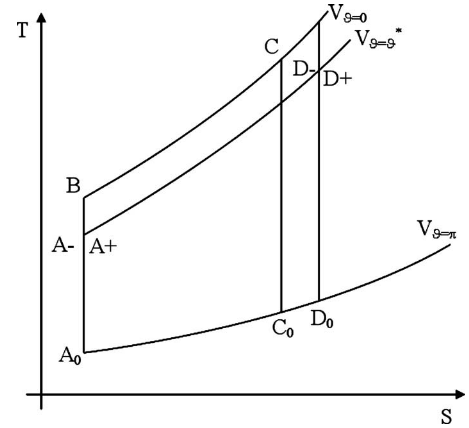

Fig. 6 Effect of instantaneous combustion phase changes on the Otto cycle

$$m \cdot \dot{Q}_w = h \cdot S' \cdot \Delta T = h(\vartheta) \cdot S'(\vartheta) \cdot (T(\vartheta) - T_w) \qquad (17)$$

where m is the gas mass, h is the convective heat exchange coefficient, S' is the combustion chamber surface, and $\dot{Q}_w = dQ_w/dt$. Both Q_{in} and Q_w are functions of the crank position ϑ, but $\dot{Q}_w$ is certainly not symmetric with respect to the TDC ($\vartheta = 0$).

In this case the differential equation cannot be solved analytically and only qualitative considerations can be made.

Taking into account, for a first approximation, the temperature progress related to the adiabatic engine (see Figs. 1 and 4), it is possible to state that the combustion CD, delayed with respect to the symmetrical one EFG, causes, for each piston position, lower gas temperatures than the advanced one AB, producing thus lower heat exchanges (Q_w) and probably reducing the whole heat subtracted from the gas during its thermodynamic cycle $Q_{2\text{total}} = Q_2 + Q_w$.

Obviously a reduction in $Q_{2\text{total}}$, at constant introduced heat Q_1, means an improvement of the cycle efficiency.

As a matter of fact, a combustion delayed by the angle ϑ with respect to the one symmetrically placed around the TDC produces an increase in ΔS_{ab} (see Eq. (4)) that means an increase in Q_2 according to Eq. (5), while, on the other hand, it produces a decrease in Q_w so when the two effects (the first tends to worsen the engine efficiency, while the second tends to improve it) compensate each other ($\Delta Q_2 = \Delta Q_w$), the optimal combustion phase is reached:

$$\Delta Q_{2\text{total}} = \Delta Q_2 + \Delta Q_w = 0$$

In the case of the heat release function (11), asymmetric with respect to the middle of the combustion arc, the above conclusions remain valid because heat transfer causes anyway the optimum condition to be forward shifted with respect to the adiabatic case.

Some quantitative conclusions can be drawn taking again into consideration the temperature progress of the adiabatic engine in the limit case of an instantaneous combustion (i.e., with $\vartheta_c = 0$).

To this purpose, Fig. 6 shows the combustion at the TDC (line BC), the advanced combustion (line A−D−), and the delayed combustion (line A+D+). The last two combustions are symmetrically placed around the TDC. In Fig. 5, the same combustion lines are represented in a plane with coordinates $[\vartheta, \zeta = Tv^{k-1}]$.

First, it is clear that an advanced or delayed instantaneous combustion brings a smaller volumetric compression ratio with respect to ρ_0 (TDC instantaneous combustion).

Second, from the analysis of Fig. 5, traced on the basis of Eq. (3), it is clear that an advanced combustion (line A−D−) produces higher mean temperatures than the delayed combustion (line A+D+), and so in the case of a delayed instantaneous combustion, the heat exchanges (Q_w) are smaller than in the case of an advanced one: This leads to the conclusion that the optimum phase of the combustion will be delayed with respect to the TDC.

When also the friction losses are taken into account, the bmep has to be considered as the engine output performance, rather than the indicated mean effective pressure (imep), being

$$\text{bmep} = \text{imep} - \text{fmep} \qquad (18)$$

where fmep is the friction mean effective pressure, which accounts for the energy drained for different reasons: the mechanical friction between piston rings and cylinder liner, the mechanical friction in the loaded bearings (crankshaft and connecting rod), and the energy required to run the auxiliary components. The maximum engine efficiency then corresponds, for a constant introduced heat Q_1, to the maximum bmep.

In Sec. 5, the optimal instantaneous combustion phase will be searched for now: For a given increment $d\vartheta^*$ of the instantaneous combustion delay (with respect to the TDC) ϑ^*, the variation $d(\text{bmep})$ will be evaluated, so that the condition $d(\text{bmep}) = 0$ will indicate the optimum delay value ϑ^*_{opt} of the instantaneous combustion.

In an ideal engine (adiabatic and without friction losses), an instantaneous combustion delayed or advanced with respect to TDC has the same effect of a volumetric compression ratio decrease on the thermodynamic cycle and its efficiency, as can be seen in Fig. 6; hence the variation in the heat subtracted from the expansion end (dQ_2), due to a given instantaneous combustion delay increment $d\vartheta^*$, can be evaluated as follows:

$$Q_1 = Q_2 \rho^{k-1}$$

For the constant Q_1 condition it follows that

$$\frac{dQ_1}{Q_1} = 0 = \frac{dQ_2}{Q_2} + (k-1)\frac{d\rho}{\rho}$$

$$\Rightarrow dQ_2 = -(k-1)\frac{d\rho}{\rho}Q_2 = -\frac{(k-1)}{\rho^{k-1}}\frac{d\rho}{\rho}Q_1 \qquad (19)$$

By definition, the imep is

$$\frac{\text{imep}}{m} = \frac{Q_1 - Q_2}{V_0} = \frac{Q_1(1 - Q_2)}{V_0} = \frac{Q_1}{V_0}\left(1 - \frac{1}{\rho^{k-1}}\right) \qquad (20)$$

where m is the in-cylinder gas mass. Concerning the real engine (i.e., in the presence of heat exchanges with the chamber walls Q_w and friction losses), the variation in imep and bmep caused by a given instantaneous combustion delay increment, at constant Q_1, can be written as

$$\frac{d(\text{imep})}{m} = \frac{d(Q_1 - Q_{2\text{total}})}{V_0} = -\frac{(dQ_2 + dQ_w)}{V_0} \qquad (21)$$

$$\frac{d(\text{bmep})}{m} = \frac{d\left(Q_1 - Q_{2\text{total}} - \frac{V_0}{m}\cdot \text{fmep}\right)}{V_0}$$

$$= -\frac{\left(dQ_2 + dQ_w + \frac{V_0}{m} d(\text{fmep})\right)}{V_0} \qquad (22)$$

and the condition of maximum efficiency, $d(\text{bmep})=0$, gives

$$dQ_2 = -dQ_w - \frac{V_0}{m} d(\text{fmep})$$

In the above relation, only the components of fmep that are actually affected by the combustion position delay must be taken into account, namely, the friction at the piston-cylinder interface and in the loaded bearings (crankshaft and connecting rod); neither the energy required by the engine accessories nor the friction caused by the inertia forces of the moving masses must be considered because these are not affected by combustion phase variations.

The two considered friction components are then fmep_1 (related to the piston-cylinder friction) and fmep_2 (related to the loaded bearings), the condition of maximum efficiency thus becomes

$$dQ_2 = -dQ_w - \frac{V_0}{m} d(\text{fmep}_1) - \frac{V_0}{m} d(\text{fmep}_2) \qquad (23)$$

The heat transferred from the gas to the chamber walls Q_w is assumed to be proportional to the exposition time of the gas to the cylinder walls and to the gas temperature (as a first approximation, the increase in heat exchange surface S' during the gas expansion is assumed to be compensated by the decrease in heat exchange coefficient h); then

$$Q_w = \text{const} \cdot \int_{\text{EC}}^{\text{BDC}} T d\vartheta$$

where T_{EC} is the gas temperature at the end of the instantaneous combustion (which remains almost unchanged for small combustion retard ϑ^*).

The increase in the heat exchanged with the chamber walls due to a delay increment $d\vartheta^*$ (with respect to the TDC) of the instantaneous combustion is then

$$dQ_w \approx -\text{const} \cdot T_{\text{EC}} \cdot d\vartheta^*$$

The last relation is quite acceptable considering that a combustion delay increment $d\vartheta^*$ around the TDC (where the in-cylinder volume is minimum) does not have a considerable effect on the temperature at the end of combustion (EC) so that $T_{\text{EC}} \approx \text{const}$ (see Figs. 5 and 6).

Since, as a first approximation, once the combustion has been completed, it can be considered as

$$TV^{k-1} = \text{const}$$

then

$$Q_w = \text{const} \cdot \int_{\text{EC}}^{\text{BDC}} T d\vartheta = \text{const} \cdot T_{\text{EC}} \int_{\text{EC}}^{\text{BDC}} \left(\frac{V^*}{V}\right)^{k-1} d\vartheta$$

where V is the instantaneous chamber volume (and V^* is the value it assumes at the end of combustion when $\vartheta = \vartheta^*$), and k is the gas isentropic coefficient. The mean crank rotation $d\vartheta$ during the expansion from the EC to the BDC can be considered, as a first approximation, proportional to the volume variation dV.

Hence

$$d\vartheta \approx \frac{\pi}{V_0} dV \Rightarrow Q_w = \text{const} \cdot T_{\text{EC}} \frac{\pi}{V_0} \int_{\text{EC}}^{\text{BDC}} \left(\frac{V^*}{V}\right)^{k-1} dV$$

being

$$\int_{\text{EC}}^{\text{BDC}} \left(\frac{V^*}{V}\right)^{k-1} dV \cong \int_{\text{TDC}}^{\text{BDC}} \left(\frac{V_{\text{TDC}}}{V}\right)^{k-1} dV \qquad (24)$$

it follows that

$$Q_w \cong \text{const} \cdot T_{\text{EC}} \frac{\pi}{V_0} \int_{\text{TDC}}^{\text{BDC}} \left(\frac{V_{\text{TDC}}}{V}\right)^{k-1} dV$$

$$= \text{const} \cdot T_{\text{TDC}} \cdot \pi \frac{\rho^{2-k} - 1}{(2-k)(\rho - 1)} \qquad (25)$$

and then

$$\frac{dQ_w}{Q_w} = -\frac{d\vartheta^*}{\pi} \frac{(2-k)(\rho-1)}{\rho^{2-k} - 1} \qquad (26)$$

As can be observed in Eq. (26), a delay increment $d\vartheta^*$ of the instantaneous combustion produces a decrease in Q_w (hence, from Eq. (21), an imep increase) and an increase in Q_2 (see Fig. 6 and Eq. (19) where $d\vartheta^* > 0$ involves $d\rho < 0$), which instead causes an imep decrease in Eq. (21); the maximum of imep then implies a null variation $d(\text{imep})$ for a given increment $d\vartheta^*$, i.e., from Eq. (21), $-dQ_w = dQ_2$.

The losses due to the piston-cylinder friction during the gas expansion fmep_1 are here supposed, as a first approximation, to be proportional to the in-cylinder pressure and to the piston stroke, and hence to the chamber volume variation:

$$\text{fmep}_1 = \text{const} \cdot \int_{\text{EC}}^{\text{BDC}} p \tan(\varphi) \cdot dV$$

where φ is the angle between the connecting rod and the cylinder axis, which is a function of the crank angle ϑ and assumes the value φ^* when the combustion delay angle is ϑ^*. If p_{EC} represents the gas pressure at the end of combustion, the increase in friction losses due to the delay increment $d\vartheta^*$ (corresponding to a volume variation dV^*) of the instantaneous combustion is

$$d(\text{fmep}_1) \approx -\text{const} \cdot p_{\text{EC}} \tan(\varphi^*) \cdot dV^*$$

It is also

$$\tan(\varphi^*) \approx \sin(\varphi^*) = \frac{1}{\mu} \cdot \sin(\vartheta^*) \approx \frac{\vartheta^*}{\mu} \Rightarrow$$

$$d(\text{fmep}_1) \approx -\text{const} \cdot p_{\text{EC}} \cdot \frac{\vartheta^*}{\mu} \cdot dV^*$$

Hence the mean value of the variable $\tan(\varphi)$ during the crank rotation from the EC to the BDC can be found as follows:

$$\overline{\tan(\varphi)} \approx \overline{\sin(\varphi)} = \frac{1}{\mu} \overline{\sin(\vartheta)} = \frac{1}{\mu} \cdot \frac{1}{\pi} \cdot \int_0^{\pi} \sin(\vartheta) d\vartheta = \frac{2}{\mu \cdot \pi}$$

Assuming, as a first approximation, an isentropic expansion from the EC to the BDC, then

$$\text{fmep}_1 = \text{const} \cdot \int_{\text{EC}}^{\text{BDC}} p \tan(\varphi) \cdot dV$$

$$\approx \text{const} \cdot p_{\text{EC}} \overline{\tan(\varphi)} \cdot \int_{\text{EC}}^{\text{BDC}} \left(\frac{V^*}{V}\right)^k dV$$

$$\approx \text{const} \cdot p_{\text{EC}} \cdot \frac{2}{\mu \cdot \pi} \cdot \int_{\text{EC}}^{\text{BDC}} \left(\frac{V^*}{V}\right)^k dV$$

where the gas pressure p_{EC} has been assumed to remain almost constant for small combustion delay ϑ^* around the TDC; under the same approximation of Eq. (24), it follows that

$$\text{fmep}_1 \approx \text{const} \cdot p_{\text{EC}} \cdot \frac{2}{\mu \cdot \pi} \cdot \int_{\text{TDC}}^{\text{BDC}} \left(\frac{V_{\text{TDC}}}{V}\right)^k dV$$

$$= \text{const} \cdot \frac{2}{\mu \cdot \pi} \cdot p_{\text{EC}} V_0 \frac{\rho^{1-k}-1}{(1-k)(\rho-1)} \qquad (27)$$

$$\frac{d(\text{fmep}_1)}{\text{fmep}_1} = -\frac{\vartheta^* \cdot \pi}{2} \cdot \frac{dV^*}{V_0} \cdot \frac{(1-k)(\rho-1)}{\rho^{1-k}-1} \qquad (28)$$

As can be observed in Eq. (28), a delay increment of the instantaneous combustion $d\vartheta^*$ (to which corresponds a volume change dV^*) produces a decrease in fmep_1: This should imply a bmep increase (from Eq. (22)); the delay increment $d\vartheta^*$ however causes also a decrease in imep (21), for the shorter working stroke of the gas after the combustion (see Eq. (19)), and consequently a decrease in bmep in Eq. (22). In conclusion the maximum bmep condition can be pursued by a null variation $d(\text{bmep})$ for a given increment $d\vartheta^*$.

In a similar way to the losses fmep_1, the friction losses fmep_2 (caused in the loaded bearings) are supposed to be proportional to the in-cylinder pressure and to the crank rotation angle during the gas expansion; hence

$$\text{fmep}_2 = \text{const} \cdot \int_{\text{EC}}^{\text{BDC}} p d\vartheta$$

$$d(\text{fmep}_2) \approx -\text{const} \cdot p_{\text{EC}} d\vartheta^*$$

where $d(\text{fmep}_2)$ is the increase in fmep_2 due to the delay increment $d\vartheta^*$ of the instantaneous combustion. Under the same hypothesis made for the evaluation of $d(\text{fmep}_1)$, it follows that

$$\text{fmep}_2 \cong \text{const} \cdot p_{\text{EC}} \int_{\text{TDC}}^{\text{BDC}} \left(\frac{V_{\text{TDC}}}{V}\right)^k d\vartheta$$

$$d\vartheta \approx \frac{\pi}{V_0} dV \Rightarrow \text{fmep}_2 = \text{const} \cdot p_{\text{EC}} \frac{\pi}{V_0} \int_{\text{TDC}}^{\text{BDC}} \left(\frac{V_{\text{TDC}}}{V}\right)^k dV$$

$$\Rightarrow \text{fmep}_2 = \text{const} \cdot p_{\text{EC}} \cdot \pi \frac{\rho^{1-k}-1}{(1-k)(\rho-1)} \qquad (29)$$

$$\Rightarrow \frac{d(\text{fmep}_2)}{\text{fmep}_2} = -\frac{d\vartheta^*}{\pi} \frac{(1-k)(\rho-1)}{\rho^{1-k}-1} \qquad (30)$$

As can be observed in Eq. (30), a delay increment $d\vartheta^*$ of the instantaneous combustion produces a decrease in fmep_2, likewise for fmep_1, with similar consequences.

The maximum engine efficiency condition (23)

$$dQ_2 = -dQ_w - \frac{V_0}{m} d(\text{fmep}_1) - \frac{V_0}{m} d(\text{fmep}_2)$$

together with Eqs. (19), (26), (28), and (30), leads to the following equation:

$$-\frac{(k-1)}{\rho^{k-1}} \frac{d\rho}{\rho} Q_1 = wQ_1 \frac{d\vartheta^*}{\pi} \frac{(2-k)(\rho-1)}{\rho^{2-k}-1} + \frac{\pi \cdot \vartheta^*}{2} v_1 \cdot (\text{imep}) \frac{dV^*}{V_0} \frac{(1-k)(\rho-1)}{\rho^{1-k}-1} \frac{V_0}{m} + v_2 \cdot (\text{imep}) \frac{d\vartheta^*}{\pi} \frac{(1-k)(\rho-1)}{\rho^{1-k}-1} \frac{V_0}{m} \qquad (31)$$

where $w = Q_w / Q_1$, $v_1 = \text{fmep}_1/\text{imep}$, and $v_2 = \text{fmep}_2/\text{imep}$.

Being

$$V_0 = V_{\text{BDC}} - V_{\text{TDC}} = V_{\text{TDC}}(\rho_0 - 1) \approx V^*(\rho - 1) \Rightarrow V^* \approx \frac{V_0}{\rho - 1}, \quad \rho = \frac{V_{\text{BDC}}}{V^*}$$

the equivalent compression ratio variation due to the instantaneous combustion phase shift is then

$$\frac{d\rho}{\rho} = -\frac{dV^*}{V^*} = -\frac{dV^*}{V_0}\frac{V_0}{V^*} = -\frac{dV^*}{V_0}(\rho-1) \Rightarrow \frac{dV^*}{V_0} = -\frac{1}{\rho-1}\frac{d\rho}{\rho} \qquad (32)$$

and remembering Eq. (9),

$$\frac{d\rho}{\rho} = -\frac{\rho-1}{2} \sin\vartheta^* \left(1 + \frac{1}{\mu}\cos\vartheta^*\right) \cdot d\vartheta^* \Rightarrow \left.\frac{d\rho}{\rho}\right)_{\text{near TDC}} = -\frac{1+\mu}{2\mu}(\rho-1) \cdot \vartheta^* \cdot d\vartheta^* \qquad (33)$$

Hence, from Eqs. (31), (20), (32), and (33), the instantaneous combustion delay, which maximizes the engine efficiency, can be evaluated by the following relation:

$$\vartheta^*_{\text{opt}} = \frac{2\mu}{\pi(1+\mu)} \frac{w \dfrac{2-k}{(k-1)(\rho^{2-k}-1)} + v_2}{\rho^{1-k} - v_1 \cdot \dfrac{\pi \cdot \vartheta^*}{2}} \qquad (34)$$

In a SI engine, approximately one-third of the initial energy of the fuel becomes available as mechanical energy and the rest is lost, one-half in the form of heat through the cylinder surfaces (cooling losses), and one-half as waste heat through the exhaust pipe. In the analysis that leads to Eq. (34) only the expansion phase is concerned; hence only about two-thirds of the total cooling losses (which take place during compression, expansion, and exhaust phases) is considered in the evaluation of coefficient w; this means that $w = 1/3 \times 2/3 = 2/9 \approx 0.22$ as a value of first approximation.

A great portion of the initial fuel energy fraction that becomes mechanical work (approximately one-third) is available as an output at the crankshaft and the rest is dissipated by friction or used by accessories. A reasonable value of mechanical efficiency for a SI engine, at wide open throttle (WOT), is around 80%, which means that $\text{fmep}_{\text{total}} = 0.2(\text{imep})$, where $\text{fmep}_{\text{total}}$ (the total friction mean effective pressure) accounts for the friction between piston and cylinder, the friction in the loaded bearings, and the energy needed to move the engine accessories [8].

In the present analysis, only the friction components influenced by a combustion phase shift are taken into account, i.e., the piston-cylinder and the bearing friction due to the gas pressure, while the friction due to the inertia forces of the moving masses can be neglected, so the sum of fmep_1 and fmep_2 is supposed to be equal to one-half of $\text{fmep}_{\text{total}}$ [8].

In a spark ignition engine the friction at the piston-cylinder interface is approximately four to five times the friction in the loaded bearings (crankshaft and connecting rod bearings) [8]; consequently it can be stated that

$$\text{fmep}_1 = 4 \cdot \text{fmep}_2 \Rightarrow v_1 = 4v_2 \qquad (35)$$

and

$$v_1 + v_2 = 0.1$$

hence $v_1 = 0.02$ and $v_2 = 0.08$.

Assuming that $k = 1.3$, $\rho = 10$, $\mu = 3.18$, and $w = 0.22$, Eq. (34) yields the results shown in Table 1. As is shown, a 22% of heat exchange with chamber walls (with respect to the introduced heat Q_1) is responsible for an instantaneous combustion optimal delay of about 7 crank angle degrees (CAD). Moreover the delay angle ϑ^*_{opt} exhibits a linear dependence on the heat ratio $w(=Q_w/Q_1)$.

Table 1 Optimal instantaneous combustion delay ϑ^*_{opt} (from Eq. (34)) for different combinations of friction and heat transfer coefficient

v_1	v_2	w	ϑ^*_{opt} CAD ATDC
0.08	0.02	0.22	8.6
0.00	0.00	0.22	7.1
0.08	0.02	0.00	1.1
0.00	0.02	0.22	8.2
0.08	0.00	0.22	7.4
0.08	0.02	0.30	11.3
0.00	0.00	0.30	9.6
0.16	0.04	0.22	10.3
0.16	0.04	0.00	2.3

As regards friction effect, a 10% loss (with respect to imep) produces a combustion optimal delay of about 1 CAD, and once more the delay angle ϑ^*_{opt} is linearly related to the sum (v_1+v_2). The two phenomena (heat transfer and friction losses) slightly influence each other (as in a real engine), even if their effects do not add. Moreover the friction losses at the piston-cylinder interface (coefficient v_1), although greater than the losses in the loaded bearings (coefficient v_2), have a minor influence on the optimal delay ϑ^*_{opt} (as can be deduced, for example, by rows 4 and 5 in Table 1). This can be explained considering that the friction forces between piston and cylinder approach zero near TDC, so the fmep_1 is nearly unaffected by an instantaneous combustion phase shift in the close proximity of TDC.

5 Evaluation of the Optimal Combustion Phase by Means of Thermodynamic Simulations

In the previous paragraph, the authors showed that if the heat release function has a symmetrical derivative the optimal combustion phase in an adiabatic engine without friction losses is symmetrical with respect to the TDC (i.e., the middle of the combustion angle must be located at the TDC). The authors also arrived to a simple expression, Eq. (34), for the evaluation of the optimal instantaneous combustion phase in a real engine, that is, in the presence of heat exchanges with chamber walls and friction losses; the equation found demonstrated a good agreement with the well known experimental evidence according to which the maximum bmep is reached when the 50% of heat released by combustion is located approximately 8 CAD ATDC, a part from engine speed, load, and mixture strength [4].

If the combustion is not considered instantaneous and its heat release rate is an asymmetrical function with respect to the middle of the combustion arc, then the optimal combustion phase can be evaluated by means of thermodynamic simulations of the compression-expansion process, using heat transfer and friction loss models (e.g., as shown in Refs. [8,9]). In this kind of simulations the start and the duration of the combustion can be set as input variables and both imep and bmep can be evaluated as output quantities.

A zero-dimensional thermodynamic model based on the first law of thermodynamics (Eqs. (16) and (17)) has been realized to predict the in-cylinder pressure and temperature during the compression and expansion phases of the engine (i.e., the constant mass phases).

The first law of thermodynamics, applied to the constant mass of gas in the cylinder, yields

$$dQ_{\text{in}} - dQ_w - pdv = du = c_v \cdot dT \tag{36}$$

where c_v is the gas specific heat at constant volume.

The gas here considered (a stoichiometric mixture of C_8H_{16} and air) is supposed to respect the perfect gas law:

$$pv = RT$$

Table 2 Engine dimensions used in the simulation

Compression ratio, ρ	10
Rod to crank ratio, μ	3.18
Cylinder bore, B (mm)	79.5
Piston stroke, s (mm)	80.5

$$k = \frac{c_p}{c_v} \Rightarrow R = c_v(k-1)$$

where $R=274.7$ J/kg K is the gas constant and k is a function of temperature obtained by Ceviz and Kaymaz [10]:

$$k = 1.338 - 6 \times 10^{-5} T_{(\text{K})} + 10^{-8} T_{(\text{K})}{}^2$$

Wall heat transfers have been taken into account by means of the Woschni model, which allows the evaluation of the instantaneous convection heat exchange coefficient h [8,11,13] that figures in Eq. (17):

$$h = F_w \cdot B^{-0,2} p^{0,8} \left[C_1 v_p + C_2 V_0 \frac{T_r}{P_r V_r}(p - p_m) \right]^{0,8} T^{-0,55} \tag{37}$$

where h is the heat exchange coefficient (W/m^2 K), F_w is the gain factor (allows to fit experimental data), B is the cylinder bore (m), T is the gas temperature (K), p is the gas pressure (kPa), v_p is the mean piston speed (m/s), V_0 is the cylinder displacement, T_r, p_r, and V_r are the gas temperature, pressure, and volume at the inlet valve closure, p_m is the motored gas pressure (evaluated by means of a polytropic law), C_1 is the first model constant (=2.28), and C_2 is the second model constant ($=3.24 \times 10^{-3}$).

The ratio $Q_w/Q_1 = w$ can be tuned by means of the gain factor F_w, and the heat released by the combustion has been assumed to follow the Wiebe function [11]

$$dQ_{\text{in}}(\vartheta) = dx(\vartheta) Q_1 \quad \text{and} \quad x(\vartheta) = 1 - e^{-ay(\vartheta)^{m+1}} \tag{38}$$

where $x(\vartheta)$ is the fraction of heat released ($\vartheta_a < \vartheta < \vartheta_b$), and $y(\vartheta) = (\vartheta - \vartheta_a)/(\vartheta_b - \vartheta_a)$ is the combustion angle fraction. $m=2$ and $a=5.33$ are the values chosen for the two constants: The latter in order to obtain an almost symmetric derivative function dx/dy with its maximum in the middle of the combustion arc; it results in $x(\vartheta_a)=0$ and $x(\vartheta_b)=0.995$.

Equations (36), (17), and (38), together with the perfect gas law, lead to the differential equation

$$\frac{dp}{d\vartheta} = -\frac{k}{V}\frac{dV}{d\vartheta} \cdot p + \frac{m}{V}(k-1)\left(\frac{dQ_{\text{in}}}{d\vartheta} - \frac{dQ_w}{d\vartheta} \right) \tag{39}$$

which, numerically solved, provide the gas pressure as a function of crank position. The engine data used in the simulation are resumed in Table 2.

The friction losses at the piston-cylinder interface (fmep_1) and in the loaded bearings (fmep_2) have been evaluated by means of a simplified model based on Rezeka and Henein's work [15] as follows:

$$\text{fmep}_1(\vartheta^*) = c_1 \cdot \int_{-180}^{+180} (1 - |\sin(\vartheta)|) \cdot p(\vartheta) \cdot |\tan(\varphi)| \cdot ds \tag{40}$$

$$\text{fmep}_2(\vartheta^*) = c_2 \cdot \int_{-180}^{+180} |\cos(\vartheta)| \cdot \frac{2 \cdot p(\vartheta)}{1 + \cos(\varphi)} \cdot d\vartheta \tag{41}$$

where c_1 and c_2 are proportionality constants depending on variables not affected by the combustion phase (geometrical data and engine speed), p is the relative gas pressure ($p = |p_{\text{gas}} - 1|$ (bar)), ds is the piston movement related to the crank rotation $d\vartheta$, and φ is

Table 3 Optimal center combustion delay evaluated by means of the thermodynamic simulation

Combustion angle CAD	$\vartheta^*_{opt,1}$ (max imep) CAD	$\vartheta^*_{opt,2}$ (max bmep) CAD
60	6.7	7.5
70	7.1	7.9
80	7.5	8.5
Mean values	7.1	8.0

the angle between connecting rod and cylinder axis, function of the crank angle ϑ.

As regards the $fmep_1$ in Eq. (40), the product $p(\vartheta)\cdot|\tan(\varphi)|$ is proportional to the normal trust between the piston and the lateral surface of the cylinder; the lubricating condition between the two surfaces can be either mixed and boundary or hydrodynamic: When the piston is in the neighborhood of the firing TDC, lubrication is mixed or boundary (maximum friction coefficient) because the relative speed between the surfaces is small. Near ± 90 deg CA ATDC instead, the lubricating conditions become hydrodynamic; hence the friction coefficient drops down significantly [15] due to the relative speed increase that means much lower friction forces. Rezeka and Henein's [15] model accounts for the two lubrication modes by means of two different correlations, while in the simplified model proposed here, both the lubrication modes are represented by means of the same term $(1-|\sin(\vartheta)|)$, which is maximum near the TDC (only mixed and boundary lubrication, and maximum friction coefficient) and minimum at ± 90 deg CA ATDC where the friction forces are supposed to be zero.

In the same way, in Eq. (41) the term $2\cdot p(\vartheta)/1+\cos(\varphi)$ is proportional to the mean friction force inside the loaded bearings, while the term $|\cos(\vartheta)|$ is proportional to the friction coefficient [15]. Now the trust discriminates between the two lubricating conditions because the relative speed between journals and bearings is almost constant.

The operative conditions chosen to perform a first comparison between the results obtained by the use of Eq. (34) and those obtained by simulations are manifold absolute pressure (MAP) =1 bar, engine speed=2000 rpm, and chamber wall temperature=200°C. In these conditions w has been set to 0.22 (by means of the gain factor F_w), while v_1 and v_2, by means of the two constants c_1 and c_2, have been set in order to respect the conditions of Eq. (35):

$$fmep_1 = 4\cdot fmep_2 \quad \text{and} \quad fmep_1 + fmep_2 = 0.1\cdot imep$$

Three different values of combustion duration $(\vartheta_b-\vartheta_a)$ have been considered in the simulations, namely, 60, 70, and 80 CAD. For each combustion duration, the start of combustion (i.e., the angle ϑ_a) has been phased in order to find both the maximum imep (which takes into account only heat exchange effect) and the maximum bmep (which instead is influenced also by friction losses). The related optimal delays of the center of the combustion arc with respect to the TDC, $\vartheta^*_{opt,1}$ and $\vartheta^*_{opt,2}$, are reported in Table 3. As can be seen, the results (mean values) are in a fine agreement with those obtained by the use of the approximated formula (34) listed in Table 1; namely, the delay due only to heat exchanges is nearly the same (i.e., 7.1 CAD, second row in Table 1), and the delay obtained taking into account also friction losses corresponds (in Table 3 $\vartheta^*_{opt,2}=8.0$ CAD≈ 8.6 CAD reported in the first row of Table 1).

Some other simulations have been carried out, considering a fixed combustion duration of 70 CAD for different engine speeds at WOT, and the results are reported in Table 4. The values of v_1 and v_2 shown here are taken from data available in literature [8] and were used to fix the values of the two constants c_1 and c_2 in Eqs. (40) and (41), while the values of the heat ratio w descend from the simulation since the gain factor F_w has been fixed to obtain $w=0.22$ at 2000 rpm and MAP=1 bar.

As is reasonable, increasing engine speed causes an increase in the sum (v_1+v_2) and a decrease in the heat ratio w.

In particular, the sum (v_1+v_2) undergoes an increase of about 70%, and this leads to an increase in $(\vartheta^*_{opt,2}-\vartheta^*_{opt,1})$ of about 1.3 CAD according to simulations and of about 1.6 CAD using the formula; however, the delay $\vartheta^*_{opt,2}$ is almost unaffected by engine speed variations since the rising friction losses are counterbalanced by the decreasing heat transfer. This behavior is confirmed by experimental results, which show that the MBT spark timing is obtained when the 50% of MFB is located approximately 8 CAD ATDC regardless of the engine and of its operative conditions [4]. Table 4 also shows a perfect agreement between the simulation results and formula results as regards the maximum imep combustion center delay $\vartheta^*_{opt,1}$: This means that the formula in Eq. (34) adequately takes into account heat transfer effects. Concerning the bmep combustion center delay $\vartheta^*_{opt,2}$, there is a small difference between the two values. Equation (34) hence slightly overestimates the effects of friction losses.

Another series of simulations has been performed for different engine loads at constant speed (3000 rpm) and combustion duration (70 CAD), whose results are exposed in Table 5. Here the values of the friction loss constants c_1 and c_2 have been chosen so as to maintain coefficients v_1 and v_2 almost unchanged since it is assumed that with decreasing manifold pressure, both $fmep_1$ and $fmep_2$ reduce in the same percentage of imep so that the ratios v_1 and v_2 are poorly influenced. On the other hand the coefficient w (calibrated by means of the gain factor F_v at 2000 rpm at WOT) increases about 16%, thus causing an angle $\vartheta^*_{opt,1}$ increase of about 1 CAD, confirmed by the use of the formula. Again, as regards the optimal combustion phase $\vartheta^*_{opt,1}$, a fine agreement was found between the simulations outputs and formula results, whereas persists the small difference of about 1 CAD in the evaluation of the combustion phase for the maximum bmep $\vartheta^*_{opt,2}$.

6 Conclusions

This paper deals with the analytical determination of the best combustion phase in a spark ignition engine. The authors first

Table 4 Optimal combustion delay angles obtained at WOT for different engine speeds (combustion arc=70 deg CA)

Engine speed (rpm)	MAP (bar)	w (Q_w/Q_1)	v_1+v_2	v_1/v_2	$\vartheta^*_{opt,1}$ (CAD) (simulation)	$\vartheta^*_{opt,2}$ (CAD) (simulation)	$\vartheta^*_{opt,1}$ (CAD) (formula)	$\vartheta^*_{opt,2}$ (CAD) (formula)
2000	1	0.220	0.100	4.00	7.1	7.9	7.1	8.6
3000	1	0.200	0.116	3.14	6.3	7.5	6.4	8.3
4000	1	0.187	0.132	3.12	6.0	7.3	6.0	8.1
5000	1	0.177	0.152	2.80	5.8	7.3	5.7	8.3
6000	1	0.170	0.172	2.58	5.5	7.6	5.5	8.6

Table 5 Optimal combustion delay angles obtained for different loads at 3000 rpm (combustion duration=70 deg CA)

Engine speed (rpm)	MAP (bar)	w (Q_w/Q_1)	v_1+v_2	v_1/v_2	$\vartheta^*_{\text{opt},1}$ (CAD) (simulation)	$\vartheta^*_{\text{opt},2}$ (CAD) (simulation)	$\vartheta^*_{\text{opt},1}$ (CAD) (formula)	$\vartheta^*_{\text{opt},2}$ (CAD) (formula)
3000	1.0	0.200	0.116	3.14	6.3	7.5	6.4	8.3
3000	0.8	0.208	0.115	3.10	6.6	7.8	6.7	8.6
3000	0.6	0.218	0.113	3.07	7.1	8.2	7.0	8.9
3000	0.4	0.232	0.110	3.01	7.2	8.2	7.4	9.3

analyzed the optimal phasing of the heat release in an ideal adiabatic engine, showing that the best phase of the combustion depends on the "shape" of the heat release rate law: If this is symmetrical with respect to its maximum, then the best spark timing is obtained when the middle point of the combustion arc is located at the TDC. The case of a not symmetrical heat release rate function is also considered and solved in the Appendix. The authors then took into consideration both the effects of friction losses and heat transfer to the wall on the optimal phasing of the instantaneous combustion, thus arriving to a formula for the calculus of the best combustion phase. The results obtained by the use of this formula under different operative conditions were then compared with the results obtained by means of zero-dimensional thermodynamic simulations performed using the Wiebe function to represent the heat release law (three different combustion durations were considered). A good agreement was found in any of the case considered.

Heat exchanges with the chamber walls, which are extremely asymmetrical and occur mainly during the expansion stroke because of the higher gas temperatures, were observed to cause a mean delay of the optimal combustion phase of the order of 6 CAD with respect to the adiabatic engine.

Friction losses due to the gas pressure are also asymmetrical and occur mainly during the expansion stroke because of the higher gas pressure. They produce a smaller delay, about 1 CAD, of the optimal combustion phase position with respect to the frictionless engine.

Nomenclature

Q = whole heat received by the gas
L = mechanical work made on the gas
U = internal energy of the gas
Q_1 = whole combustion specific released by combustion
c_p = constant pressure specific heat of the gas
c_v = constant volume specific heat of the gas
ϑ = generic crank angle
ϑ_a = crank angle corresponding to combustion start
ϑ_b = crank angle corresponding to combustion end
ϑ_c = combustion arc
ϑ^* = instantaneous combustion delay with respect to the TDC
ϑ^*_{opt} = optimal instantaneous combustion delay with respect to the TDC
Q_{in} or $Q_{\text{in}}(\vartheta)$ = specific heat released by combustion at the crank position ϑ
T = gas temperature
T_w = chamber wall temperature
$V(\vartheta)$ = combustion chamber (function of the crank position ϑ)
m = constant mass of the gas in the cylinder
v = gas specific volume
p = gas pressure
S = gas specific entropy
R = gas constant
k = isentropic coefficient
c_p = constant pressure specific heat of the gas
Q_2 = heat subtracted from the gas, at constant volume, as needed to close the thermodynamic cycle
$x(\vartheta)$ = heat fraction released, during the crank rotation from ϑ_a to ϑ, with respect to the total $Q_1=Q_{\text{in}}(\vartheta_b)$
y = $(\vartheta-\vartheta_a)/\vartheta_c$ fraction of crank rotation angle
ρ or ρ_0 = volumetric compression ratio of the engine
η = thermodynamic cycle efficiency
V_0 = engine displacement
μ = rod to crank ratio
Q_w = heat subtracted to the gas by the chamber walls
$S'(\vartheta)$ = surface of the combustion chamber
h = convection heat exchange coefficient
$Q_{2\text{total}}$ = the whole heat subtracted to the gas, sum of the Q_2 and the Q_w $\zeta=Tv^{k-1}$
fmep = friction mean effective pressure
fmep_1 = friction mean effective pressure due to the friction between piston and cylinder
fmep_2 = friction mean effective pressure due to the friction in the loaded bearings
$\text{fmep}_{\text{total}}$ = $\text{fmep}_1+\text{fmep}_2$
imep = indicated mean effective pressure
T_{TDC} = gas temperature at TDC
T_{EC} = gas temperature at the end of combustion
V_{TDC} = cylinder volume at TDC
p_{TDC} = gas pressure at TDC
φ = angle between connecting rod and cylinder axis
φ^* = the angle φ when $\vartheta=\vartheta^*$
w = Q_w/Q_1
v_1 = $\text{fmep}_1/\text{imep}$
v_2 = $\text{fmep}_2/\text{imep}$
B = cylinder bore
u_m = mean gas velocity
s = piston stroke
n = engine speed
ϑ^*_1 = optimum instantaneous combustion delay considering heat exchanges
ϑ^*_2 = optimum instantaneous combustion delay considering both heat exchanges and friction losses

Appendix

The determination of the optimal combustion phase for the adiabatic and frictionless engine consists in the minimization of the entropy variation ΔS_{ab} of Eq. (4):

$$\Delta S_{ab} = \int_a^b \left[c_v \frac{dT}{T} + R\frac{dv}{v} \right] = c_v \left[\ln\frac{T_b}{T_a} + \frac{R}{c_v}\ln\frac{v_b}{v_a} \right] = c_v \ln\frac{T_b v_b^{k-1}}{T_a v_a^{k-1}}$$
$$= c_v \ln\frac{\zeta_b}{\zeta_a} = c_v \ln\left[1 + \frac{1}{c_v T_a v_a^{k-1}} \int_{\vartheta_a}^{\vartheta_b} v^{k-1} \frac{dQ_{\text{in}}}{d\vartheta} d\vartheta \right]$$
$$= c_v \ln\left[1 + \frac{1}{c_v T_M v_M^{k-1}} \int_{\vartheta_a}^{\vartheta_b} v^{k-1} \frac{dQ_{\text{in}}}{d\vartheta} d\vartheta \right] \quad \text{(A1)}$$

where T_M and v_M represent the gas temperature and specific volume at the BDC; hence Eq. (A1) becomes

$$\Delta S_{ab} = c_v \ln\left[1 + \frac{1}{c_v T_{\text{BDC}} \rho_0^{k-1}} \int_{\vartheta_a}^{\vartheta_b} \left(\frac{v}{v_{\text{TDC}}} \right)^{k-1} \frac{dQ_{\text{in}}}{d\vartheta} d\vartheta \right] \quad \text{(A2)}$$

Now, if the combustion is phased so as to put its middle point at the TDC (i.e., $\vartheta_a = -0.5\vartheta_c$ and $\vartheta_b = +0.5\vartheta_c$) then the determination of the optimal combustion phase with respect to piston motion (hence to in-cylinder volume) merely consists in the minimization of the following integral:

$$\int_{\vartheta_a}^{\vartheta_b} \left(\frac{V(\vartheta')}{V_{\text{TDC}}} \right)^{k-1} \frac{dQ_{\text{in}}}{d\vartheta} d\vartheta \quad \text{(A3)}$$

where $\vartheta' = \vartheta - \lambda$, with λ the phase difference between the combustion middle point ($\vartheta = 0$) and the TDC position ($\vartheta' = 0$). If the following functions f, q, and F are taken into consideration,

$$f = \left(\frac{V(\vartheta')}{V_{\text{TDC}}} \right)^{k-1} = \left(\frac{V(\vartheta - \lambda)}{V_{\text{TDC}}} \right)^{k-1} = f(\vartheta;\lambda)$$

$$q = \frac{dQ_{\text{in}}}{d\vartheta} = \text{const} \cdot \frac{dx(\vartheta)}{d\vartheta} = q(\vartheta) \quad \text{(A4)}$$

$$F = f \cdot q = F(\vartheta;\lambda)$$

the integral to be minimized in Eq. (A3) becomes

$$\int_{\vartheta_a}^{\vartheta_b} f\ q\ d\vartheta = \int_{\vartheta_a}^{\vartheta_b} F\ d\vartheta = \int_{\vartheta_a}^{\vartheta_b} F(\vartheta;\lambda) d\vartheta \quad \text{(A5)}$$

According to the Leibniz rule [16],

$$\frac{\delta}{\delta\lambda} \left(\int_{\vartheta_a}^{\vartheta_b} F(\vartheta;\lambda) d\vartheta \right) = \int_{\vartheta_a}^{\vartheta_b} \frac{\delta F}{\delta\lambda} d\vartheta \quad \text{(A6)}$$

which, being the function q independent of the variable λ, becomes

$$\int_{\vartheta_a}^{\vartheta_b} \frac{\delta F}{\delta\lambda} d\vartheta = \int_{\vartheta_a}^{\vartheta_b} \frac{\delta(f\ q)}{\delta\lambda} d\vartheta = \int_{\vartheta_a}^{\vartheta_b} q \frac{\delta f}{\delta\lambda} d\vartheta = 0 \quad \text{(A7)}$$

From Eq. (A7) it follows the important conclusion that if the function q (i.e., the heat release rate) is symmetrical with respect to the middle of the combustion arc, the optimal combustion phase is obtained when $\lambda = 0$, i.e., when the combustion is centered around the TDC, because the derivative $\delta f/\delta\lambda$ is antisymmetric with respect to the TDC.

In the more general case of function q not symmetrical with respect to its middle point, the integral in Eq. (A7) can be solved analytically or numerically. A fast solution can be, however, obtained by means of some approximations; taking into consideration the following series expansions $\cos\vartheta \approx 1 - 0.5\vartheta^2$ and $(1 \pm \varepsilon)^a \approx 1 \pm a\varepsilon$, the in-cylinder volume can be expressed as $V(\vartheta')/V_{\text{TDC}} \approx 1 - C \cdot \vartheta'^2 = 1 + \varepsilon$ and the function f becomes

$$f = \left(\frac{V}{V_{\text{TDC}}} \right)^{k-1} \approx 1 + (k-1) \cdot \left(\frac{V}{V_{\text{TDC}}} - 1 \right) \approx 1 + C' \cdot \vartheta'^2$$
$$= 1 + C'(\vartheta - \lambda)^2 \Rightarrow \frac{\delta f}{\delta\lambda} \approx -2 \cdot C' \cdot \vartheta' = -2 \cdot C'(\vartheta - \lambda) \quad \text{(A8)}$$

where C and C' are constants.

Then the integral in Eq. (A7) becomes

$$\int_{\vartheta_a}^{\vartheta_b} q \frac{\delta f}{\delta\lambda} d\vartheta \approx \int_{-0.5\vartheta_c}^{0.5\vartheta_c} qC'' \cdot (\vartheta - \lambda) d\vartheta = 0$$
$$\Rightarrow \int_{-0.5\vartheta_c}^{0.5\vartheta_c} (\vartheta - \lambda) \frac{dx}{d\vartheta} d\vartheta = 0 \quad \text{(A9)}$$

which gives the optimal phase lag λ:

$$\lambda = \frac{\int_{-0.5\vartheta_c}^{0.5\vartheta_c} \vartheta \frac{dx}{d\vartheta} d\vartheta}{\int_{-0.5\vartheta_c}^{0.5\vartheta_c} \frac{dx}{d\vartheta} d\vartheta} = \int_{-0.5\vartheta_c}^{0.5\vartheta_c} \vartheta \frac{dx}{d\vartheta} d\vartheta = \vartheta_g \quad \text{(A10)}$$

Equation (A10) shows that the phase lag λ coincides with the centroid ϑ_g of the area subtended by the function $dx/d\vartheta$. As example, if the Wiebe function (11) represents the heat release rate, the optimum combustion phase will then be obtained with a slight advance with respect to the symmetric position.

Up to now, the entropy variation during the combustion phase has been evaluated supposing the gas specific heat c_v to remain constant; it is instead a function of the gas temperature and composition and may change appreciably, thus introducing a further asymmetrical element in the evaluation of the minimum ΔS_{ab} combustion phase.

Considering the following temperature dependence $c_v = B + A \cdot T$ (where A and $B = c_{v0} - T_0 A$ are two constants) and $k_0 = (c_{v0} + R')/c_{v0}$, Eq. (1) becomes

$$dQ_{\text{in}} = TdS = c_v dT + pdv = BdT + T\left(R\frac{dv}{v} + AdT \right) \quad \text{(A11)}$$

Assuming that $\varphi(\vartheta) = (R/B)(1/v)(dv/d\vartheta) + (A/B)(dT/d\vartheta)$ and $\psi(\vartheta) = -(1/B)(dQ_{\text{in}}/d\vartheta)$, Eq. (2) and its general solution remain valid, Eq. (3) hence becomes

$$Tv^{k_0-1} e^{AT/B} = T_a v_a^{k_0-1} e^{AT_a/B} + \frac{1}{B} \int_{\vartheta_a}^{\vartheta} v^{k_0-1} e^{AT/B} \frac{dQ_{\text{in}}}{d\vartheta} d\vartheta \quad \text{(A12)}$$

and the entropy variation across the combustion phase is now

$$\Delta S_{ab} = B \ln\left[1 + \frac{1}{BT_a v_a^{k_0-1} e^{AT_a/B}} \int_{\vartheta_a}^{\vartheta_b} v^{k_0-1} e^{AT/B} \frac{dQ_{\text{in}}}{d\vartheta} d\vartheta \right]$$
$$= B \ln\left[1 + \frac{1}{BT_M v_M^{k_0-1}} \int_{\vartheta_a}^{\vartheta_b} v^{k_0-1} e^{A(T-Ta)/B} \frac{dQ_{\text{in}}}{d\vartheta} d\vartheta \right] \quad \text{(A13)}$$

As shown, it mainly differs from Eq. (4) for the function $e^{A(T-Ta)/B}$ inside the integral, which is a rising function of the temperature difference $(T - T_a)$, which, in turn, increases when the combustion is advanced (see, for example, Fig. 1, the advanced combustion AB compared with the retarded one CD): That is the reason why, when the c_v variation with temperature is considered, the optimal combustion phase moves in retard of about 1 CAD, thus reducing the optimal spark advance.

References

[1] Pestana, G. W., 1989, "Engine Control Methods Using Combustion Pressure Feedback," SAE Paper No. 890758.

[2] Plint, M., and Martyr, A., 1999, *Engine Testing—Theory and Practice*, 2nd ed., Butterworth & Heinemann, Oxford.

[3] Eriksson, L., 1999, "Spark Advance Modeling and Control," Ph.D. thesis, Linköping University, Linköping, Sweden, www.vehicular.isy.liu.se.

[4] Pipitone, E., 2008, "A Comparison Between Combustion Phase Indicators for Optimal Spark Timing," ASME J. Eng. Gas Turbines Power, **130**(5), p. 052808, DOI: 10.1115/1.2939012.

[5] Powell, J. D., 1993, "Engine Control Using Cylinder Pressure: Past, Present and Future," ASME J. Dyn. Syst., Meas., Control, **115**, pp. 343–350.

[6] Bargende, M., 1995, "Most Optimal Location of 50% Mass Fraction Burned and Automatic Knock Detection Components for Automatic Optimization of SI-Engine Calibrations," Motortech. Z., **56**, pp. 632–638.

[7] Matekunas, F. A., Battiston, P. A., Chang, C. F., Sellnau, M. C., and Lancaster, D. R., 2000, "Cylinder-Pressure-Based Engine Control Using Pressure-Ratio Management and Low-Cost Non-Intrusive Cylinder Pressure Sensor," SAE Paper No. 2000-01-0932.

[8] Ferguson, C. R., and Kirkpatrick, A. T., 2001, *Internal Combustion Engines Applied Thermosciences*, Wiley, New York.

[9] Ramos, J. I., 1989, *Internal Combustion Engine Modelling*, Hemisphere, New York.

[10] Ceviz, M. A., and Kaymaz, I., 2005, "Temperature and Air–Fuel Ratio Dependent Specific Heat Ratio Functions for Lean Burned and Unburned Mixture," Energy Convers. Manage., **46**, pp. 2387–2404.

[11] Heywood, J. B., 1988, *Internal Combustion Engines Fundamentals*, McGraw-Hill Automotive Technology Series, McGraw-Hill, New York.

[12] Heywood, J. B., 1994, "Combustion and Its Modeling in Spark-Ignition Engines," International Symposium COMODIA 94, Yokohama, Japan, Jul.

[13] Finol, C. A., and Robinson, K., 2006, "Thermal Modelling of Modern Engines: A Review of Empirical Correlations to Estimate the In-Cylinder Heat Transfer Coefficient," Proc. Inst. Mech. Eng., Part D (J. Automob. Eng.), **220**, pp. 1765–1781.

[14] Horlock, J. H., and Winterbone, D. E., 1986, *The Thermodynamics and Gas Dynamics of Internal Combustion Engines*, Vol. II, Clarendon, Oxford.

[15] Rezeka, S. F., and Henein, N. A., 1984, "A New Approach to Evaluate Instantaneous Friction and Its Components in Internal Combustion Engines," International Congress and Exposition, Detroit, MI, Feb. 27–Mar. 2, SAE Paper No. 840179.

[16] Korn, G. A., and Korn, T. M., 1968, *Mathematical Handbook for Scientists and Engineers*, McGraw-Hill, New York.